Calculus with Vectors

Additional Proofs

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1) Theorem 5 i) on page 22(?). In this proof we want to show that $\vec{u}+\vec{w}=\vec{w}+\vec{u}$ when $\vec{u}$ and $\vec{w}$ are in $\mathbb{R}^n$.

PROOF. Here we have

$$\begin{aligned}\vec{u}\cdot\vec{w} &= (u_1+w_1, u_2+w_2, \dots, u_n+w_n)\\ &= (w_1+u_1, w_2+u_2, \dots, w_n+u_n)\\ &= \vec{w}+\vec{u}\ .\end{aligned}$$

□

2) Theorem 6 ii) on page 23(?). In this proof we want to show thatfor any vector $\vec{v}\in\mathbb{R}^n$ we have $\vec{v}\cdot+\vec{v}=0$ if and only if $\vec{v}=\vec{0}$.

PROOF. Here we have

$$\begin{aligned}\vec{u}\cdot\vec{w} &= (u_1+w_1, u_2+w_2, \dots, u_n+w_n)\\ &= (w_1+u_1, w_2+u_2, \dots, w_n+u_n)\\ &= \vec{w}+\vec{u}\ .\end{aligned}$$

□

3) Theorem 7 i) on page 23(?). In this proof we want to show that $\vec{u}\cdot\vec{w}=\vec{w}\cdot\vec{u}$ when $\vec{u}$ and $\vec{w}$ are in $\mathbb{R}^n$.

PROOF. Here we have

$$\begin{aligned}\vec{u}\cdot\vec{w} &= u_1w_1+u_2w_2+\cdots+u_nw_n\\ &= w_1u_1+w_2u_2+\cdots+w_nu_n\\ &= \vec{w}\cdot\vec{u}\ .\end{aligned}$$

□

4) Theorem 9 on page 36(?). From scalar to vector for (i).

We want to show that if $\{veca_n\}$ and $\{vecb_n\}$ converge to $\vec{L}$ and $\vec{M}$ respectively, then $\lim_{n\to\infty}\left(\vec{a}_n+\vec{b}_n\right)=\vec{L}+\vec{M}$. The only difference in this proof from the scalar proof is that we use the triangle inequality for vectors, $\|\vec{a}+\vec{b}\|\le\|\vec{a}\|+\|\vec{b}\|$.

PROOF. Fix an $r > 0$. Choose N and P such that if $n > N$ and $p > P$ then $|\vec{a}_n - \vec{L}| < r/2$ and $|\vec{b}_n - \vec{M}| < r/2$. Let R be the larger of N and P. If $n > R$, then

$$\begin{aligned} \|(a_n + b_n) - (L + M)\| &= \|(a_n - L) + (b_n - M)\| \\ &\leq \|(a_n - L)\| + \|(b_n - M)\| \\ &< r/2 + r/2 \\ &< r. \end{aligned}$$

By our definition, $\{\vec{a}_n + \vec{b}_n\}$ converges to $L + M$. □

5) Theorem 10 on page 37(?).
6) Theorem 11 on page 37(?). Fill in details.
7) Theorem 12 on page 38(?).
8) Theorem 13 on page 38(?)
9) Theorem 18 on page 44(?)
10) Theorem 19 on page 44(?)
11) Theorem 24 on page 55(?)
12) Theorem 27 on page 74(?)
13) Theorem 28 on page 77(?)